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PORE SIZE DISTRIBUTION IN SIMULATION OF MASS TRANSPORT IN POROUS MEDIA: A CASE STUDY IN RESERVOIR ANALYSIS

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ABSTRACT

The modeling and numerical simulation of mass transport in porous media is discussed in this work by using the so-called pore size distribution for computing transport properties. The pore-size distribution is a property of the pore structure of a porous medium. This can be used to estimate the different transport properties, amongst other, the permeability. By starting with a formula for the absolute permeability, the simulation of water and oil transport in reservoirs is considered by solving mass conservation equations with the help of the control volume method. The influence of the pore size distribution on the transport behavior is discussed to demonstrate the adequacy of the use of pore size distribution in studying the behavior of reservoirs.

Keywords: pore size distribution, mass transfer, reservoir, control volume method, numerical simulation.

1. INTRODUCTION

The modeling and numerical simulation of mass transport in porous media is a topic of great interest in different fields of engineering and have attracted the attention of research institutions for decades. The mass transport of liquids in general and of water and oil in particular finds its application not only in civil engineering, chemical engineering, food processing and pharmaceuticals but also in cutting edge technologies like in electronic packaging [1-9]. Many works were realized to study the behavior of porous media under various transport conditions and for different fluids. Amongst these, a huge amount of effort was put on the modelling and simulation of water and oil transport in reservoirs [10-12]. In doing so, one of the difficulties engineers face in simulating transport phenomena inside porous media is how to compute the transport properties of a porous medium. This difficulty appears when the transport equations at pore level are up-scaled to continuum level in order to establish a system of continuum equations describing the different transport phenomena in a porous body. In theory, these transport phenomena can be directly described and analyzed at pore level. However, the problem becomes very large and difficult to solve when we consider practical cases. This is an obstacle even in the age of super computers and parallel computing. In using continuum models for the analysis of transport phenomena in porous media, a porous medium is considered as continuous with averaged (effective) transport properties. These properties must be measured experimentally before being used in numerically simulation. They are functions of pore level properties of each particular porous material. In order to understand how the properties of material pore structure influence the transport behavior, one interesting approach is to take into account the size and the distribution of the pores of porous materials. In what follow, we discuss the continuum model for mass transport in reservoir simulation. We will also discuss a model that can be used to compute one of the most important transport parameters, namely the absolute permeability for use in the continuum model. By making use of this model, we will present a numerical example in which the influence of the pore size distribution on the mass transfer of water and oil is examined with the help of the control volume method.

2. CONTINUUM MODEL OF MASS TRANSPORT IN RESERVOIR SIMULATION

We consider here the transport of mass in a porous medium in which two components, namely water and oil, are present. We assume that both water and oil are in liquid form and during the whole process, they remain as liquid. The governing equations of the system oil-water can be derived by considering the mass, momentum and energy conservation equation of oil and water. We will limit ourselves in this work to quasi-isothermal processes and therefore assume that the conservation of energy is satisfied automatically. In what follows, we will then discuss the conservation equations of mass and momentum.

Without going into detailed derivation, the mass conservation equation for water in liquid phase can be written in the format

$$\frac{\partial}{\partial t}\left(\psi S_w \rho_w\right) - \nabla \cdot \left[\rho_w \frac{k_{rw}}{\eta_w} K \cdot \nabla p_w\right] - q_w = 0, \tag{1}$$

where ψ is the porosity of the porous body under consideration, S_w the saturation of water, ρ_w the density of water, p_w the pressure of water, η_w the viscosity of water, k_{rw} the relative permeability of water, q_w the flux of water and K the absolute permeability of the porous body. The absolute permeability (also called intrinsic permeability) K is a measure for the ability of a fluid to flow through a medium, when a single fluid is present in the medium. The absolute permeability is fluid independent and depends only on the structure of the porous material. The relative permeability k_{rw} describes how permeability is reduced due to the presence of a second phase. The relative permeability depends on the saturation of the fluids.

In the same way, we can write the mass conservation equation for oil in liquid phase as

$$\frac{\partial}{\partial t}\left(\psi S_o \rho_o\right) - \nabla \cdot \left[\rho_o \frac{k_{ro}}{\eta_o} K \nabla p_o\right] - q_o = 0, \tag{2}$$

where S_o is the saturation of oil, ρ_o the density of oil, p_o the pressure of oil, η_o the viscosity of oil, k_{ro} the relative permeability of oil, and q_o the flux of oil.

The conservation of momentum for our problem can be described by using the generalized Darcy’s law, which can be formulated as

$$\mathbf{v}_w = -\frac{k_{rw} K}{\eta_w}\left(\nabla p_w - \nabla \Psi_w\right) \quad \text{and} \quad \mathbf{v}_o = -\frac{k_{ro} K}{\eta_o}\left(\nabla p_o - \nabla \Psi_o\right) \tag{3}$$

where $\mathbf{v}_w$ and $\mathbf{v}_o$ are the mass average velocities of water and oil, Ψ_w and Ψ_o are the corresponding gravity potentials. In many cases, the gravity effect is small and can be ignored.

Note that the saturations of the two phases (water and oil) should follow the requirement that their sum is unity

$$S_w + S_o = 1 \tag{4}$$

Besides, the existence of the so-called capillary pressure p_c means that there is a difference between the pressure of the two phases (water and oil)

$$p_c = p_o - p_w \tag{5}$$

In order to solve the above system of equations, the following material properties need to be determined: absolute permeability, relative permeability, porosity, viscosity and capillary pressure. In the next Section, we will consider the particular task of determining the absolute permeability of a porous medium by considering its porous structure.

3. PORE SIZE DISTRIBUTION APPROACH FOR COMPUTING ABSOLUTE PERMEABILITY

In order to compute the absolute permeability, we make use of the model presented by Metzger and Tsotsas [13]. In this model, different capillary tubes are set perpendicular to the exchange surface of the porous body and the solid phase is arranged in parallel (bundle of capillaries). The model is one-dimensional since it is assumed that there is no lateral resistance to heat or mass transfer between the solid and capillaries, hence local thermal equilibrium is fulfilled. We restrict ourselves to large enough pore sizes so that for every capillary the boundary between liquid phases can be described by a meniscus having a capillary pressure. In order to see how the permeability of a porous medium can be computed, let us consider one capillary which is fully saturated by water. On the one hand, the volumetric flow rate is calculated from the Poiseille's equation.

$$\dot{V} = \frac{1}{8\eta_w} \cdot \frac{\Delta p_w}{L} \cdot \pi \cdot r^4 \tag{6}$$

where L is the capillary length, r the capillary radius. On the other hand, the mean velocity v (volumetric flow rate per total cross section of porous medium) of the liquid can be described by the generalized Darcy law. In this calculation, we assume that gravitational effects are negligible and that velocity is small enough to neglect inertial effects. If we apply Darcy law to a fully saturated capillary (k_{rw} = 1), we obtain

$$\bar{v} = \frac{\mathrm{K}}{\eta_w} . \frac{\Delta p_w}{L} \tag{7}$$

By comparing Eqs. (6) and (7) the absolute permeability can be found to be

$$K = \frac{1}{8} r^2 \tag{8}$$

an extension to the bundle of capillaries yields

$$K = \frac{1}{8} \int_{r_{\min}}^{r_{\max}} r^2 \frac{dV}{dr} dr \tag{9}$$

where the interval [r_{min}, r_{max}] is the total range of the pore size distribution.

In the next Section, we will use this formula in our numerical simulation to investigate the influence of the micro-structure of a porous medium on its transport behavior.

4. NUMERICAL RESULTS

We consider in this section a reservoir problem (Figure 1), in which the upper and low boundaries of the domain under consideration are impermeable (no flow boundaries). Oil comes from the left-hand side and water flows out from the right-hand side of the domain. The subdomain in the middle has significantly lower absolute permeability (K_2) in comparison with the rest of the domain (K_1). By considering the change of K_1 as function of pore-size distribution of the porous medium in the outer domain, we want to examine how the transport of oil from the left-hand side is affected by the change of pore size and its distribution. In our analysis, the whole domain is initially saturated with liquid water (S_w = 1, S_o = 0) with initial water pressure of 5 bar ($p_w = 5.10^5$Pa). Water is extracted from the right-hand side of the domain at the rate of 50 g.m^{-1}.s^{-1}. As oil comes in from left-hand side boundary, the pressure of oil at this boundary is set at 5 bar ($p_o = 5.10^5$ Pa). The porosity of the whole domain is assumed to be $\psi = 0{,}2$. For the analysis, the absolute permeability of the small domain is selected to be of approximately one order of magnitude smaller than the rest of the structure: $K_2 = 10^{-9}$ m^2 and will be kept constant. The absolute permeability of the rest of the domain is computed by assuming 4 cases in which the pores have different sizes and distributions as presented in Table 1. The absolute permeability is computed using the formulas presented in Section 3.

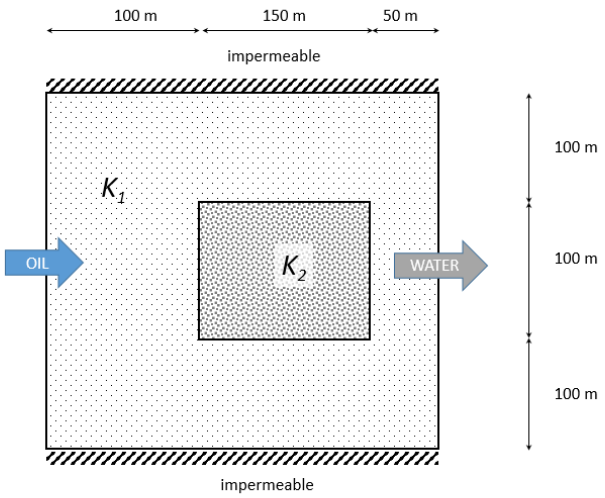
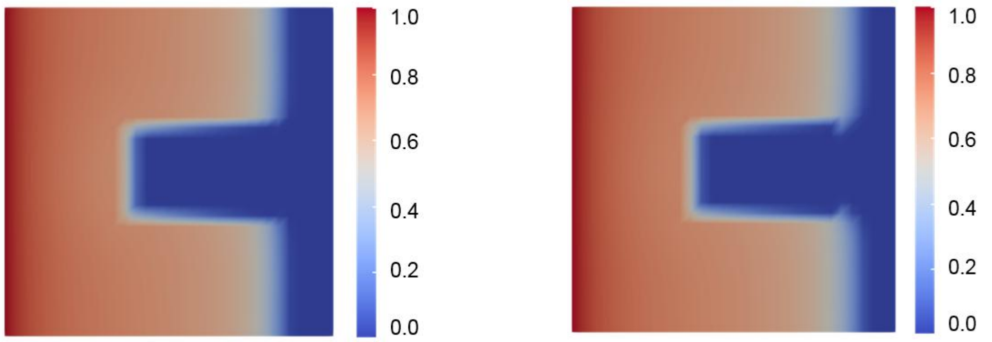


Figure 1. Reservoir problem.

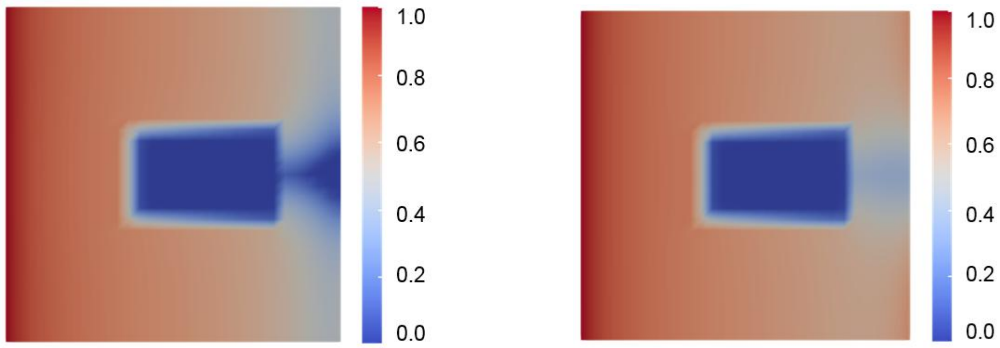
Table 1. Absolute permeability K_1 with different pore size distributions.

	Pore radius (μm)	Pore size distribution (μm)	Absolute permeability (m^2)
Case 1	1000	± 100	8.559×10^{-8}
Case 2	1000	± 250	1.246×10^{-7}
Case 3	2000	± 200	3.423×10^{-7}
Case 4	2000	± 500	4.983×10^{-7}



Case 1: 1000 ± 100 μm   Case 2: 1000 ± 250 μm

Figure 2. Saturation of oil S_o with small pore radius (r = 1000μm).



Case 3: 2000 ± 200 μm   Case 4: 2000 ± 500 μm

Figure 3. Saturation of oil S_o with large pore radius (r = 2000 μm).

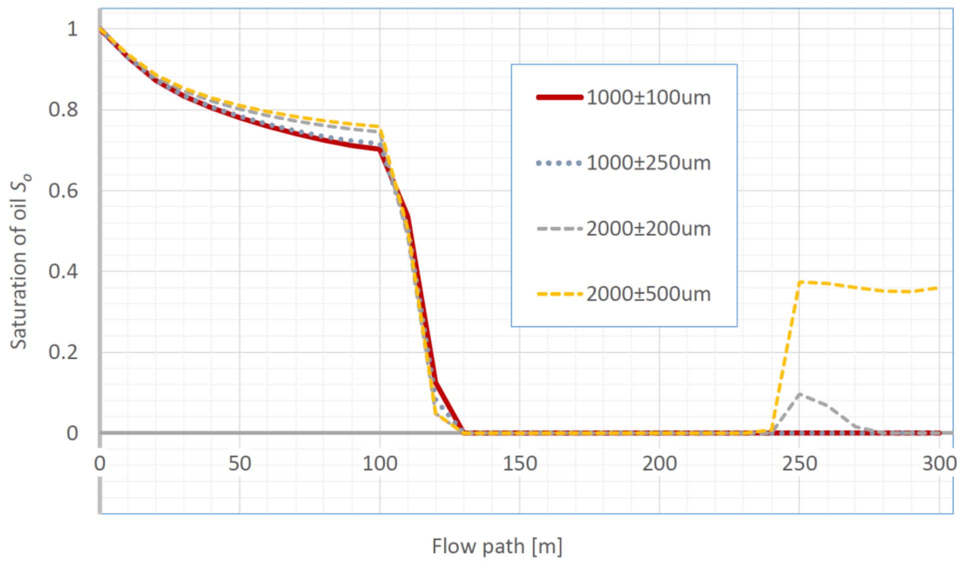


Figure 4. Saturation of oil S_o along middle flow path with different pore radii and distributions.

The simulation is realized using the control volume method to solve the system of equations presented in Section 2. The results are presented in Figures 2, 3 and 4. On Figures 2 and 3, the saturation of oil after 500000 seconds (approximately 6 days) is presented for different pore sizes and distributions. On Figure 4 the same saturation but along the middle flow path is presented. It can be observed that the size and the distribution of the pores can have significant impact on the flow of oil into the domain under consideration. With larger pores, more oil can be transported into the domain. The same is true for larger distribution.

5. CONCLUSION

In this work, the modelling and numerical simulation of mass transport in porous media are discussed for the case of oil transport in reservoir. The absolute permeability of the domain of the reservoir is computed by considering the properties of the pore structure of the domain's material. This is possible thank to the so-called "bundle of capillaries" model. The model with bundle of capillaries is applied to compute the change in absolute permeability and correspondingly the change in transport behavior of the material of the reservoir. The numerical results show that not only the size of the pores but also the distribution of the pore size can have significant impact on the transport behavior of the reservoir.

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